Time-Evolution of a Collective Meson Field by the use of a Squeezed State

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Abstract

A time-evolution of quantum meson fields is investigated in a linear sigma model by means of the time-dependent variational approach with a squeezed state. The chiral condensate, which is a mean field of the quantum meson fields, and quantum fluctuations around it are treated self-consistently in this approach. The attention is payed to the description of the relaxation process of the chiral condensate, where the energy stored in the mean field configuration flows to the fluctuation modes. It is shown that the quantum fluctuations plays an important role to describe this relaxation process.

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§1. Introduction

One of the recent interests associated with the physics of the relativistic heavy-ion collisions is the investigation of the dynamics of matter at very high energy density. As for the chiral phase transition, especially, the possible formation of a disoriented chiral condensate in ultra-relativistic nuclear collisions has been discussed. [1)‐10)] The time-evolution of the chiral condensate in the chiral phase transition is a main subject to study in diverse work, namely, the dynamics of the phase transition is investigated. In these studies, the O(4) sigma model has been used mainly as an effective model of quantum chromodynamics (QCD). The time-evolution of a scalar field is also important to describe some aspects of inflationary model of universe. [11)‐16)] However, most analytical results in this field concern the classical equations of motion.

When the high energy nucleus-nucleus collisions occur, one expects the formation of the quark-gluon plasma. Also, it is expected that the chiral phase transition occurs and the chiral symmetric phase is realized. After the collisions, the hot region created by the collision will expand spatially and the temperature will become lower. Thus, the chiral phase transition will occurs again and the chiral broken phase will be realized. Here, let us consider so-called quench scenario. The chiral condensate which is a chiral order parameter must grow from 0 to $\langle \sigma \rangle$. The relaxation process should be realized since the energy stored initially should flow elsewhere. Thus, the interesting and important problem is to make it clear how the relaxation occurs. The dissipative process in the linear sigma model has been treated in the framework of the Caldeira-Leggett theory [9)] in the context of the decay of the disoriented chiral condensate. In their work, the mechanism of relaxation has been given within the classical level.

One of the present authors (Y.T.) together with D. Vautherin and T. Matsui has indicated that the quantum fluctuation plays an important role for the chiral phase transition [17)] and the relaxation of the collective isospin rotation of the chiral condensate. [18)] The Gaussian approximation of the functional Schrödinger picture [19)] for the field theory has been adopted in our previous work. In this approach, both the mean field and the quantum fluctuations can be treated self-consistently including the quantum effect of the higher order of $\hbar$. This approach is essentially same as the time-dependent variational approach with a squeezed state. [20)] In this paper, we will use the later approach to investigate the time-evolution of the chiral condensate and the quantum fluctuation around it because the fluctuation modes can be constructed explicitly. Except for the previous work, [17), 18)] the time-development of the chiral condensate was examined to leading order in a large N expansion in Ref.6) by Cooper et. al. However, in our time-dependent variational approach with a squeezed

state, it is easily possible to investigate the role of the quantum fluctuations systematically because the quantum state is definitely constructed by means of the squeezed state and the mode-expansion for quantum fluctuations is explicitly performed. The main aim of this paper is to point out that the quantum fluctuations are responsible for the occurrence of the relaxation process of the chiral condensate or chiral order parameter. Of course, the expansion of the system leads to a friction term.[8), 10)] However, even if the expansion is not taken into account, it will be shown that the behavior like the damped oscillation for the chiral condensate appears due to the inclusion of the quantum fluctuations.

This paper is organized as follows : The time-dependent variational approach with a squeezed state is reviewed briefly and the mode functions in this approach are introduced in the case of the uniform condensate in the next section. This approach is applied to the linear sigma model and the basic equations are obtained in §3. The expanding geometry is presented in Appendix. The numerical results and discussion are given in §4. The last section is devoted to a summary.

§2. Time-dependent variational approach with a squeezed state

In this section we briefly give the basic ingredients in the time-dependent variational approach with a squeezed state.[20)] The main advantage of this approach to the dynamical problem including both the degrees of freedom of the mean field (order parameter) and the quantum fluctuations around it ("mesons" if quantized) is in the fact that both degrees of freedom can be treated self-consistently through the nonlinear coupling. Further, we can get the canonical equations of motion describing their time-development.

2.1. *Basic equations of motion*

Apart from the sigma model, for simplicity, let us first consider a scalar field theory with one component. We start with the following Hamiltonian :

$$\hat{H} = \int d^3\boldsymbol{x} \left\{ \frac{1}{2}\pi(\boldsymbol{x})^2 + \frac{1}{2}\left(\nabla\phi(\boldsymbol{x})\right)^2 + U[\phi(\boldsymbol{x})] \right\} \ . \tag{2·1}$$

We take a following squeezed state as a trial state :

$$|\Phi(t)\rangle = \exp\left(S(t)\right) \cdot N(t) \exp\left(T(t)\right) |0\rangle \ , \tag{2·2}$$

$$S(t) = i \int d^3\boldsymbol{x}[\overline{\pi}(\boldsymbol{x},t)\phi(\boldsymbol{x}) - \overline{\varphi}(\boldsymbol{x},t)\pi(\boldsymbol{x})] \ ,$$

$$T(t) = \int\int d^3\boldsymbol{x}d^3\boldsymbol{y} \ \phi(\boldsymbol{x})\left[-\frac{1}{4}(G^{-1}(\boldsymbol{x},\boldsymbol{y},t) - G^{(0)}(\boldsymbol{x},\boldsymbol{y})^{-1}) + i\Sigma(\boldsymbol{x},\boldsymbol{y},t)\right]\phi(\boldsymbol{y}) \ ,$$

where $|0\rangle$ is the reference vacuum with respect to free boson annihilation operators a_k; $a_k|0\rangle = 0$, and $G^{(0)}(\boldsymbol{x}, \boldsymbol{y}) = \langle 0|\phi(\boldsymbol{x})\phi(\boldsymbol{y})|0\rangle$. Further, N is a normalization factor. The variational functions are $\overline{\varphi}(\boldsymbol{x}, t)$, $\overline{\pi}(\boldsymbol{x}, t)$, $G(\boldsymbol{x}, \boldsymbol{y}, t)$ and $\Sigma(\boldsymbol{x}, \boldsymbol{y}, t)$, respectively. The expectation values for the field operators are obtained as

$$\langle \Phi(t)|\phi(\boldsymbol{x})|\Phi(t)\rangle = \overline{\varphi}(\boldsymbol{x}, t) , \qquad \langle \Phi(t)|\pi(\boldsymbol{x})|\Phi(t)\rangle = \overline{\pi}(\boldsymbol{x}, t) ,$$

$$\langle \Phi(t)|\phi(\boldsymbol{x})\phi(\boldsymbol{y})|\Phi(t)\rangle = \overline{\varphi}(\boldsymbol{x}, t)\overline{\varphi}(\boldsymbol{y}, t) + G(\boldsymbol{x}, \boldsymbol{y}, t) ,$$

$$\langle \Phi(t)|\pi(\boldsymbol{x})\pi(\boldsymbol{y})|\Phi(t)\rangle = \overline{\pi}(\boldsymbol{x}, t)\overline{\pi}(\boldsymbol{y}, t) + \frac{1}{4}G^{-1}(\boldsymbol{x}, \boldsymbol{y}, t) + 4\langle \boldsymbol{x}|\Sigma(t)G(t)\Sigma(t)|\boldsymbol{y}\rangle ,$$

$$(2\cdot3)$$

where we used the following notation :

$$\langle \boldsymbol{x}|\Sigma(t)G(t)\Sigma(t)|\boldsymbol{y}\rangle \equiv \int \int d^3\boldsymbol{x}' d^3\boldsymbol{y}' \Sigma(\boldsymbol{x}, \boldsymbol{x}', t)G(\boldsymbol{x}', \boldsymbol{y}', t)\Sigma(\boldsymbol{y}', \boldsymbol{y}, t) . \qquad (2\cdot4)$$

Here, $\overline{\varphi}(\boldsymbol{x}, t)$ represents the condensate or the mean field. The two-point function $G(\boldsymbol{x}, \boldsymbol{y}, t)$ means the two-point correlation function with the order of $\hbar$. This function represents the quantum fluctuation around the mean field $\overline{\varphi}(\boldsymbol{x}, t)$. For later convenience, we introduce $\mathcal{M}^{(n)}[\overline{\varphi}(\boldsymbol{x}, t)]$ defined as

$$\mathcal{M}^{(n)}[\overline{\varphi}(\boldsymbol{x}, t)] = \exp\left\{\frac{1}{2}G(\boldsymbol{x}, \boldsymbol{x}, t)\frac{\partial^2}{\partial z^2}\right\} U^{(n)}[z]\Bigg|_{z=\overline{\varphi}(\mathbf{x}, t)} \qquad (2\cdot5)$$

with $U^{(n)}[z] = d^n U/dz^n$. Here, $\mathcal{M}^{(0)}[\overline{\varphi}(\boldsymbol{x}, t)]$ is nothing but the expectation value of the functional $U[\phi(\boldsymbol{x})]$ of the field operator $\phi(\boldsymbol{x})$. From this expression, it is seen that the fluctuation around the mean field can be taken into account with the loop contributions $G(\boldsymbol{x}, \boldsymbol{x}, t)$.

The state $|\Phi(t)\rangle$ is determined as an functional of $\overline{\varphi}(\boldsymbol{x}, t), \overline{\pi}(\boldsymbol{x}, t), G(\boldsymbol{x}, \boldsymbol{y}, t)$ and $\Sigma(\boldsymbol{x}, \boldsymbol{y}, t)$, and their time-development is governed by the time-dependent variational principle :

$$\delta \int dt \langle \Phi(t)|i\frac{\partial}{\partial t} - \hat{H}|\Phi(t)\rangle = 0 . \qquad (2\cdot6)$$

The expectation value of the Hamiltonian is expressed as

$$H = \langle \Phi(t)|\hat{H}|\Phi(t)\rangle = \int d^3\boldsymbol{x} \mathcal{E}(\boldsymbol{x}) , \qquad (2\cdot7)$$

$$\mathcal{E}(\boldsymbol{x}) = \frac{1}{2}\overline{\pi}(\boldsymbol{x}, t)^2 + \frac{1}{2}\left(\nabla\overline{\varphi}(\boldsymbol{x}, t)\right)^2 + \mathcal{M}^{(0)}[\overline{\varphi}(\boldsymbol{x}, t)]$$

$$+ \frac{1}{8}\langle \boldsymbol{x}|G^{-1}(t)|\boldsymbol{x}\rangle + 2\langle \boldsymbol{x}|\Sigma(t)G(t)\Sigma(t)|\boldsymbol{x}\rangle + \frac{1}{2}\lim_{\mathbf{y}\to\mathbf{x}}\nabla_{\mathbf{x}}\nabla_{\mathbf{y}}\langle \boldsymbol{x}|G(t)|\boldsymbol{y}\rangle .$$

The variational equation (2·6) gives the canonical equations of motion :

$$\frac{\partial \overline{\varphi}(\boldsymbol{x}, t)}{\partial t} = \frac{\delta H}{\delta \overline{\pi}(\boldsymbol{x}, t)} , \qquad \frac{\partial \overline{\pi}(\boldsymbol{x}, t)}{\partial t} = -\frac{\delta H}{\delta \overline{\varphi}(\boldsymbol{x}, t)} ,$$

$$\frac{\partial G(\boldsymbol{x}, \boldsymbol{y}, t)}{\partial t} = \frac{\delta H}{\delta \Sigma(\boldsymbol{x}, \boldsymbol{y}, t)} , \qquad \frac{\partial \Sigma(\boldsymbol{x}, \boldsymbol{y}, t)}{\partial t} = -\frac{\delta H}{\delta G(\boldsymbol{x}, \boldsymbol{y}, t)} . \tag{2·8}$$

As a result, we obtain the following equations of motion :

$$\frac{\partial^2 \overline{\varphi}(\boldsymbol{x}, t)}{\partial t^2} - \nabla^2 \overline{\varphi}(\boldsymbol{x}, t) + \mathcal{M}^{(1)}[\overline{\varphi}(\boldsymbol{x}, t)] = 0 , \tag{2·9}$$

$$\left\{ -\nabla_{\boldsymbol{x}}^2 + \mathcal{M}^{(2)}[\overline{\varphi}(\boldsymbol{x}, t)] \right\} G(\boldsymbol{x}, \boldsymbol{y}, t) = \frac{1}{4} \langle \boldsymbol{x} | G^{-1}(t) | \boldsymbol{y} \rangle - 2 \langle \boldsymbol{x} | \dot{\Sigma}(t) G(t) | \boldsymbol{y} \rangle$$
$$- 4 \langle \boldsymbol{x} | \Sigma(t)^2 G(t) | \boldsymbol{y} \rangle , \tag{2·10}$$

$$\frac{\partial G(\boldsymbol{x}, \boldsymbol{y}, t)}{\partial t} = 2[\langle \boldsymbol{x} | G(t) \Sigma(t) | \boldsymbol{y} \rangle + \langle \boldsymbol{x} | \Sigma(t) G(t) | \boldsymbol{y} \rangle] , \tag{2·11}$$

where we used the equation $\partial_t \overline{\varphi} = \overline{\pi}$. The above-derived equations of motion are the basic equations in the squeezed state approach.

2.2. Mode expansion

From now on, let us concentrate our attention on the equations of motion for quantum fluctuations in Eqs.(2·10) and (2·11). We assume that the condensate is uniform spatially with a translational invariance. With this assumption, the two-point functions $G(\boldsymbol{x}, \boldsymbol{y}, t)$ and $\Sigma(\boldsymbol{x}, \boldsymbol{y}, t)$ only depend on $\boldsymbol{x} - \boldsymbol{y}$. Thus it is possible to carry out the Fourier transformation as follows :

$$G(\boldsymbol{x}, \boldsymbol{y}, t) = \frac{1}{(2\pi)^3} \int d^3 \boldsymbol{k} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{y})} G_{\mathbf{k}}(t) , \quad \Sigma(\boldsymbol{x}, \boldsymbol{y}, t) = \frac{1}{(2\pi)^3} \int d^3 \boldsymbol{k} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{y})} \Sigma_{\mathbf{k}}(t) . \tag{2·12}$$

From Eq.(2·11) the equation $\dot{G}_{\mathbf{k}} = 4 G_{\mathbf{k}} \Sigma_{\mathbf{k}}$ is obtained. By using the above transformation and Eq.(2·10), we can get the following equation of motion for the Fourier modes $G_{\mathbf{k}}(t)$:

$$\frac{1}{2} \ddot{G}_{\mathbf{k}}(t) - \frac{\dot{G}_{\mathbf{k}}(t)^2}{4 G_{\mathbf{k}}(t)} + \boldsymbol{k}^2 G_{\mathbf{k}}(t) - \frac{1}{4 G_{\mathbf{k}}(t)} + [\mathcal{M}^{(2)} G]_{\mathbf{k}} = 0 , \tag{2·13}$$

where $[\mathcal{M}^{(2)} G]_{\mathbf{k}}$ means the Fourier-transformed mode of $\mathcal{M}^{(2)}[\overline{\varphi}(t)] G(\boldsymbol{x}, \boldsymbol{y}, t)$: $\mathcal{M}^{(2)}[\overline{\varphi}(t)] G(\boldsymbol{x}, \boldsymbol{y}, t) = (1/(2\pi)^3) \int d^3 \boldsymbol{k} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{y})} [\mathcal{M}^{(2)} G]_{\mathbf{k}}$. It should be noted here that $G_{\mathbf{k}}$ has the meaning of the Gaussian width in terms of the functional Schrödinger picture, [19), 20)] that is, $G_{\mathbf{k}}$ is positive definite. We can thus express $G_{\mathbf{k}}(t)$ as a square of the mode function $\eta_{\mathbf{k}}$:

$$G_{\mathbf{k}}(t) = \eta_{\mathbf{k}}(t)^2 . \tag{2·14}$$

Substituting Eq.(2·14) into Eq.(2·13), the equation of motion for fluctuation mode is obtained as

$$\ddot{\eta}_{\mathbf{k}}(t) + \boldsymbol{k}^2 \eta_{\mathbf{k}}(t) + [\mathcal{M}^{(2)} G]_{\mathbf{k}} \frac{1}{\eta_{\mathbf{k}}(t)} - \frac{1}{4\eta_{\mathbf{k}}(t)^3} = 0 \; . \qquad (2\cdot15)$$

The energy density in Eq.(2·7) is expressed in terms of the mean field $\overline{\varphi}(t)$ and the mode functions $\eta_{\mathbf{k}}(t)$ as follows :

$$\mathcal{E}(\boldsymbol{x}) = \frac{1}{2} \left(\frac{\partial \overline{\varphi}(t)}{\partial t} \right)^2 + \mathcal{M}^{(0)}[\overline{\varphi}(t)] + \frac{1}{(2\pi)^3} \int d^3\boldsymbol{k} \left\{ \frac{1}{2}\dot{\eta}_{\mathbf{k}}^2 + \frac{1}{2}\boldsymbol{k}^2 \eta_{\mathbf{k}}^2 + \frac{1}{8\eta_{\mathbf{k}}^2} \right\} \; . \qquad (2\cdot16)$$

Here, we give two comments : One is about the mode expansion. If we rewrite the mode function into $\eta_{\mathbf{k}}(t) = 1/\sqrt{2\epsilon_{\mathbf{k}}(t)}$, the two-point correlation function $G(\boldsymbol{x}, \boldsymbol{y}, t)$ is expressed as $G(\boldsymbol{x}, \boldsymbol{y}, t) = \sum_{\mathbf{k}} \xi_{\mathbf{k}}(\boldsymbol{x}, t)\xi_{\mathbf{k}}^*(\boldsymbol{y}, t)/(2\epsilon_{\mathbf{k}}(t))$ where $\xi_{\mathbf{k}}(\boldsymbol{x}, t) = e^{i\mathbf{k}\cdot\mathbf{x}} e^{-i\int^t \epsilon_{\mathbf{k}}(t')dt'}$. This expression is nothing but that of the usual mode-expansion by the plane wave. Here $\epsilon_{\mathbf{k}}(t)$ corresponds to the time-dependent single particle energy. Another is the treatment of the system at finite temperature. If it is necessary to deal with the system at finite temperature in the thermal equilibrium state, the mode expansion must be slightly modified. In the thermal equilibrium, the variables are time-independent. The field operator is divided into $\phi = \overline{\varphi} + \hat{\phi}$. Further, $\hat{\phi}$ is expanded by the mode functions :

$$\hat{\phi} = \sum_{\mathbf{k}} \frac{1}{\sqrt{2\epsilon_{\mathbf{k}}}} (\xi_{\mathbf{k}} b_{\mathbf{k}} + \xi_{\mathbf{k}}^* b_{\mathbf{k}}^\dagger) \; , \qquad (2\cdot17)$$

where $b_{\mathbf{k}}|\Phi\rangle = 0$ at zero temperature. In the thermal equilibrium state, we assume that $\langle b_{\mathbf{k}} b_{\mathbf{k}'} \rangle = \langle b_{\mathbf{k}}^\dagger b_{\mathbf{k}'}^\dagger \rangle = 0$ and $\langle b_{\mathbf{k}}^\dagger b_{\mathbf{k}'} \rangle = n_{\mathbf{k}} \delta_{\mathbf{k}\mathbf{k}'}$ where $n_{\mathbf{k}}$ is the bose distribution function : $n_{\mathbf{k}} = 1/(e^{\beta\epsilon_{\mathbf{k}}} - 1)$. Then, the two-point function $G(\boldsymbol{x}, \boldsymbol{y})$ is expressed as

$$G(\boldsymbol{x}, \boldsymbol{y}) = \sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}}} [\xi_{\mathbf{k}}(\boldsymbol{x})\xi_{\mathbf{k}}^*(\boldsymbol{y}) + (\xi_{\mathbf{k}}(\boldsymbol{x})\xi_{\mathbf{k}}^*(\boldsymbol{y}) + \xi_{\mathbf{k}}^*(\boldsymbol{x})\xi_{\mathbf{k}}(\boldsymbol{y}))n_{\mathbf{k}}] \; . \qquad (2\cdot18)$$

Thus, it is necessary to replace the Fourier transformation in Eq.(2·14) into

$$G_{\mathbf{k}} = (1 + 2n_{\mathbf{k}})\eta_{\mathbf{k}}^2 \qquad (2\cdot19)$$

when we treat the system at finite temperature in the thermal equilibrium state.

§3. Squeezed state approach to the linear sigma model

In this section, let us derive the equations of motion for the linear sigma model in this approach developed in §2. The Hamiltonian density is given as

$$\mathcal{H} = \frac{1}{2}\pi_a(\boldsymbol{x})^2 + \frac{1}{2}\nabla\phi_a(\boldsymbol{x})\nabla\phi_a(\boldsymbol{x}) + \lambda\left(\phi_a(\boldsymbol{x})^2 - v^2\right)^2 - h\phi_0(\boldsymbol{x}) \qquad (3\cdot1)$$

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where a runs $0 \sim N-1$. The index 0 means the sigma-field and $1 \sim N-1$ mean the pi-fields. Hereafter, we introduce $m^2 = 4\lambda v^2$. The trial state we here adopt is expressed as

$$|\Phi(t)\rangle = \prod_{a=0}^{N-1} \exp\{S_a(t)\} \cdot N_a(t) \exp\{T_a(t)\}|0\rangle . \tag{3.2}$$

The expectation value of Hamiltonian density, $\langle\Phi(t)|\mathcal{H}|\Phi(t)\rangle$, is easily calculated similar to Eq.(2.7). For later convenience, we define $\mathcal{M}_a^{(n)}$ same as Eq.(2.5) :

$$\mathcal{M}_a^{(n)}[\overline{\varphi}_a(\boldsymbol{x},t)] = \exp\left\{\frac{1}{2}G_a(\boldsymbol{x},\boldsymbol{x},t)\frac{\partial^2}{\partial z^2}\right\} \cdot \frac{d^n}{dz^n}\left(-\frac{m^2}{2}z^2 + \lambda z^4\right)\bigg|_{z=\overline{\varphi}_a(\mathbf{x},t)} \tag{3.3}$$

with $a = 0 \sim N-1$. We omit the constant term $m^4/(16\lambda)$ because of no contribution to the dynamics.

The time-dependent variational principle (2.6) gives us the canonical equations of motion for $(\overline{\varphi}_a, \overline{\pi}_a)$ and (G_a, Σ_a) and leads us to the following equations :

$$\frac{\partial\overline{\varphi}_a(\boldsymbol{x},t)}{\partial t} = \frac{\delta H}{\delta\overline{\pi}_a(\boldsymbol{x},t)} = \overline{\pi}_a(\boldsymbol{x},t) ,$$

$$\frac{\partial\overline{\pi}_a(\boldsymbol{x},t)}{\partial t} = -\frac{\delta H}{\delta\overline{\varphi}_a(\boldsymbol{x},t)}$$

$$= -\left[-\nabla^2\overline{\varphi}_a(\boldsymbol{x},t) + \mathcal{M}_a^{(1)}[\overline{\varphi}_a(\boldsymbol{x},t)]\right.$$

$$\left. + 4\lambda\overline{\varphi}_a(\boldsymbol{x},t)\sum_{b\neq a}\left(\overline{\varphi}_b(\boldsymbol{x},t)^2 + G_b(\boldsymbol{x},\boldsymbol{x},t)\right) - h\delta_{a0}\right] , \tag{3.4}$$

$$\frac{\partial G_a(\boldsymbol{x},\boldsymbol{y},t)}{\partial t} = \frac{\delta H}{\delta\Sigma_a(\boldsymbol{x},\boldsymbol{y},t)} = 2\left[\langle\boldsymbol{x}|G_a(t)\Sigma_a(t)|\boldsymbol{y}\rangle + \langle\boldsymbol{x}|\Sigma_a(t)G_a(t)|\boldsymbol{y}\rangle\right] ,$$

$$\frac{\partial\Sigma_a(\boldsymbol{x},\boldsymbol{y},t)}{\partial t} = -\frac{\delta H}{\delta G_a(\boldsymbol{x},\boldsymbol{y},t)}$$

$$= -\left[\frac{1}{2}\mathcal{M}_a^{(2)}[\overline{\varphi}_a(\boldsymbol{x},t)]\delta(\boldsymbol{x}-\boldsymbol{y}) - \frac{1}{8}\langle\boldsymbol{x}|\frac{1}{G_a(t)^2}|\boldsymbol{y}\rangle + 2\langle\boldsymbol{x}|\Sigma_a(t)^2|\boldsymbol{y}\rangle\right.$$

$$\left. + \frac{1}{2}\nabla_{\boldsymbol{x}}\nabla_{\boldsymbol{y}}\delta(\boldsymbol{x}-\boldsymbol{y}) + 2\lambda\sum_{b\neq a}\left(\overline{\varphi}_b(\boldsymbol{x},t)^2 + G_b(\boldsymbol{x},\boldsymbol{x},t)\right)\delta(\boldsymbol{x}-\boldsymbol{y})\right] . \tag{3.5}$$

The last equation in Eq.(3.5) is simply expressed by multiplying $G_a(\boldsymbol{x},\boldsymbol{y},t)$ from the right-hand side :

$$\left\{-\nabla_{\boldsymbol{x}}^2 + \mathcal{M}_a^{(2)}[\overline{\varphi}_a(\boldsymbol{x},t)]\right\}G_a(\boldsymbol{x},\boldsymbol{y},t)$$

$$= \frac{1}{4} \langle \boldsymbol{x}|G_a^{-1}(t)|\boldsymbol{y}\rangle - 2\langle \boldsymbol{x}|\dot{\Sigma}_a(t)G_a(t)|\boldsymbol{y}\rangle - 4\langle \boldsymbol{x}|\Sigma_a(t)^2 G_a(t)|\boldsymbol{y}\rangle$$

$$-4\lambda \sum_{b\neq a} \left(\overline{\varphi}_b(\boldsymbol{x},t)^2 + G_b(\boldsymbol{x},\boldsymbol{y},t)\right) G_a(\boldsymbol{x},\boldsymbol{y},t) \ . \tag{3.6}$$

Let us consider the case where the system has translational invariance. Then, the condensates do not depend on space-coordinate $\boldsymbol{x}$. Further, the two-point functions such as $G_a(\boldsymbol{x},\boldsymbol{y},t)$ depend on the difference $\boldsymbol{x} - \boldsymbol{y}$ only. Then, we can carry out the Fourier transformation in the same way as those in the section 2. Comparing Eq.(2·10) with Eq.(3·6), we here regard $\mathcal{M}^{(2)}[\overline{\varphi}]$ in Eq.(2·10) as $\mathcal{M}_a^{(2)}[\overline{\varphi}_a] + 4\lambda \sum_{b\neq a}[\overline{\varphi}_b^2 + G_b(t)]$. Performing the mode expansion similar to Eqs.(2·12) and (2·14), we finally obtain the following equations of motion for the condensate (mean field) and the quantum fluctuations :

$$\ddot{\overline{\varphi}}_a(t) - m^2 \overline{\varphi}_a(t) + 4\lambda \overline{\varphi}_a(t)^3 + 12\lambda \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \, \eta_{\boldsymbol{k}}^a(t)^2 \cdot \overline{\varphi}_a(t)$$

$$+4\lambda \sum_{b\neq a} \left(\overline{\varphi}_b(t)^2 + \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \, \eta_{\boldsymbol{k}}^b(t)^2\right) \overline{\varphi}_a(t) - h\delta_{a0} = 0 \ ,$$

$$\ddot{\eta}_{\boldsymbol{k}}^a(t) + \left[\boldsymbol{k}^2 - m^2 + 12\lambda\overline{\varphi}_a(t)^2 + 12\lambda \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \, \eta_{\boldsymbol{k}'}^a(t)^2 \right.$$

$$\left. +4\lambda \sum_{b\neq a} \left(\overline{\varphi}_b(t)^2 + \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \, \eta_{\boldsymbol{k}'}^b(t)^2\right)\right] \eta_{\boldsymbol{k}}^a(t) - \frac{1}{4\eta_{\boldsymbol{k}}^a(t)^3} = 0 \ . \tag{3.7}$$

These are the basic equations in the linear sigma model in the squeezed state approach. The condensate and the quantum fluctuations are coupled each other and the time-evolution of both the degrees of freedom should be determined self-consistently.

It is instructive to deal with the static case in the O(4) linear sigma model. The differential terms with respect to time t disappear in this case. Let us assume that $\overline{\varphi}_1 = \overline{\varphi}_2 = \overline{\varphi}_3 = 0$, that is, the chiral condensate points in the sigma-direction. Also, it is assumed that the each fluctuation mode of the direction of pi is identical, which we denote $\eta^\pi : \eta_{\boldsymbol{k}}^1 = \eta_{\boldsymbol{k}}^2 = \eta_{\boldsymbol{k}}^3 = \eta_{\boldsymbol{k}}^\pi$. Then, the equations (3·7) read

$$\left(-m^2 + 4\lambda\overline{\varphi}_0^2 + 12\lambda \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \, \eta_{\boldsymbol{k}}^{0\,2} + 12\lambda \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \, \eta_{\boldsymbol{k}}^{\pi\,2}\right) \overline{\varphi}_0 - h = 0 \ ,$$

$$\left(\boldsymbol{k}^2 + M_\sigma^2\right) \eta_{\boldsymbol{k}}^0 = \frac{1}{4\eta_{\boldsymbol{k}}^{0\,3}} \ ,$$

$$\left(\boldsymbol{k}^2 + M_\pi^2\right) \eta_{\boldsymbol{k}}^\pi = \frac{1}{4\eta_{\boldsymbol{k}}^{\pi\,3}} \ , \tag{3.8}$$

where we define the sigma meson mass M_σ and the pion mass M_π as

$$M_\sigma^2 = -m^2 + 12\lambda\overline{\varphi}_0^2 + 12\lambda \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \, \eta_{\boldsymbol{k}'}^{0\,2} + 12\lambda \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \, \eta_{\boldsymbol{k}'}^{\pi\,2} \ ,$$

$$M_\pi^2 = -m^2 + 4\lambda\overline{\varphi}_0^2 + 4\lambda \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \, \eta_{\boldsymbol{k}'}^{0\,2} + 20\lambda \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \, \eta_{\boldsymbol{k}'}^{\pi\,2} \ . \qquad (3\cdot9)$$

This is identical with the results derived in the previous paper.[17] (The parameters $\lambda/24$ and m_0^2 should be replaced into λ and $-m^2$, respectively, in this paper.) From Eq.(3·8), $\eta_{\boldsymbol{k}}^{0\,2}$ and $\eta_{\boldsymbol{k}}^{\pi\,2}$ are expressed as

$$\eta_{\boldsymbol{k}}^{0\,2} = \frac{1}{2\sqrt{\boldsymbol{k}^2 + M_\sigma^2}} \ , \qquad \eta_{\boldsymbol{k}}^{\pi\,2} = \frac{1}{2\sqrt{\boldsymbol{k}^2 + M_\pi^2}} \qquad (3\cdot10)$$

and the two point function G_0 and G_π are also obtained as

$$G_0 = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{1}{2\sqrt{\boldsymbol{k}^2 + M_\sigma^2}} \ , \qquad G_\pi = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{1}{2\sqrt{\boldsymbol{k}^2 + M_\pi^2}} \ . \qquad (3\cdot11)$$

The equations (3·8)∼(3·10) compose the set of self-consistent equations.

§4.   Numerical result and discussion

We investigate the time-evolution of the mean field and quantum fluctuations around it based on the above-derived equations of motion in the context of the quench scenario in the relativistic nucleus-nucleus collision as was mentioned in §1. Our attention is paid to describe the damping of the amplitude of the mean field. It will be shown numerically that the fluctuation modes are responsible for the damping of the mean field configuration. Namely, since the energy stored in the mean field configuration flows to the fluctuation modes, the behavior like the damped oscillation appears in the mean field configuration. Our aim is here to describe the damping behavior qualitatively.

In general, the renormalization procedure is necessary in our approach because the loop contribution is included through the function $G(\boldsymbol{x}, \boldsymbol{x}, t)$. The renormalization will be carried out because the linear sigma model is renormalizable. However, it should be noted that the sigma model is here regarded as a low energy effective model of QCD. Thus, we become free from the complexity of the renormalization due to introducing the momentum cutoff in the context of the low energy effective theory.

The model parameters are given so as to reproduce the pion mass ($M_\pi = 138$ MeV), the sigma meson mass ($M_\sigma = 600$ MeV) and the pion decay constant ($f_\pi = 92$ MeV) in the
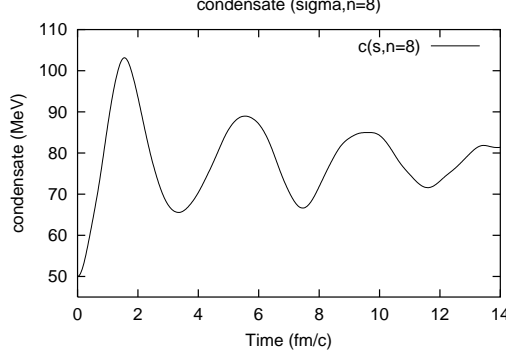
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Fig. 1. The time-evolution of the chiral condensate is depicted in which the fluctuation modes are contained up to $n^2 = 8^2$.

static case, respectively, that is,

$$m^2 = \frac{M_\sigma^2 - 3M_\pi^2}{2} + \frac{3(M_\sigma^2 - M_\pi^2)G_\pi}{f_\pi^2 + G_0 - G_\pi} \ ,$$

$$\lambda = \frac{M_\sigma^2 - M_\pi^2}{8(f_\pi^2 + G_0 - G_\pi)} \ ,$$

$$h = \frac{M_\pi^2 f_\pi^3 + M_\sigma^2 f_\pi(G_0 - G_\pi)}{f_\pi^2 + G_0 - G_\pi} \ , \tag{4.1}$$

where G_0 and G_π have been defined in Eq.(3·11) and can be calculated with an appropriate three-momentum cutoff. In general, the initial condition of the mean field is taken arbitrary in our framework. However, the quantum fluctuation modes should be taken so that the quantum effects are as small as possible in static case. Thus, we adopt the initial conditions of the quantum fluctuation modes so as to satisfy $\partial E_{\rm fl}/\partial \eta_{\bf k}^a = 0$ with $\overline{\varphi}_0 = 92$ MeV, where $E_{\rm fl}$ represents the energy of the quantum fluctuation part. Also, we take $\dot{\eta}_{\bf k}^a(t = t_0) = 0$.

Let us investigate the time-evolution of the mean field on the σ-direction only. We assume $\overline{\varphi}_i = 0$ with $i = 1 \sim 3$. In numerical calculation, we adopt the box normalization with the spatial length being L for each direction. We then impose the periodic boundary condition for the fluctuation modes, namely, the allowed values of momenta are $k_x = (2\pi/L)n_x$ and so on, where n_x is integer. The fluctuation modes labeled by (n_x, n_y, n_z) are included in each isospin direction up to $n^2 = n_x^2 + n_y^2 + n_z^2 = 8^2$. This corresponds to the momentum cutoff $\Lambda \sim 1$ GeV (990 MeV) since we have adopted the collisional region as $L^3 = 10^3$ fm^3. It should be here noted that the equations of motion in Eq.(3·7) has the time-reversal invariance. Thus, the amplitude of the oscillation of the chiral condensate becomes large again even if the behavior of the damped oscillation is realized. However, in the realistic situation, the quantized pions and/or sigma mesons are emitted with a certain energy. Therefore, in our approach, it is enough to investigate the behavior of the condensate before the amplitude increases again.
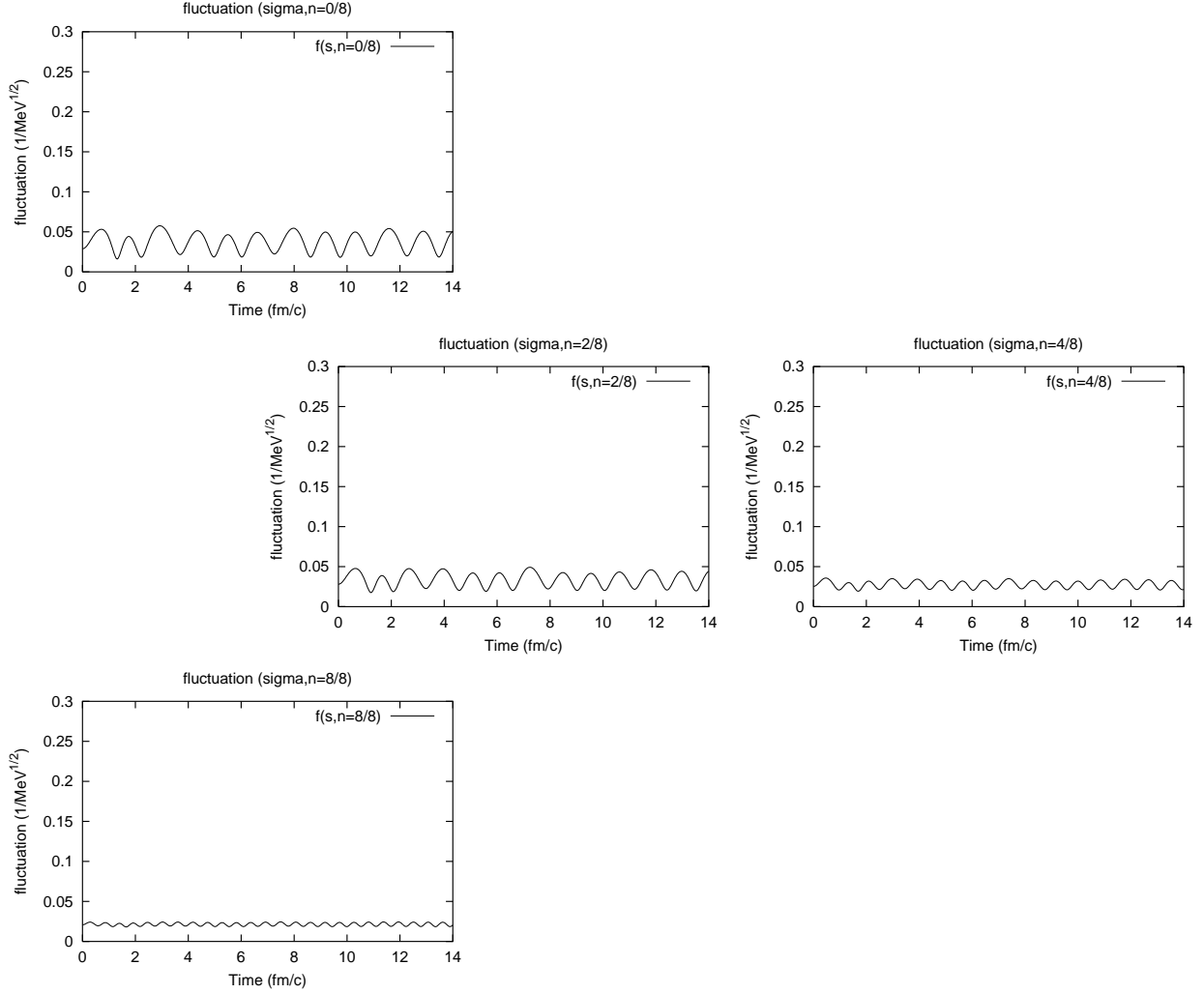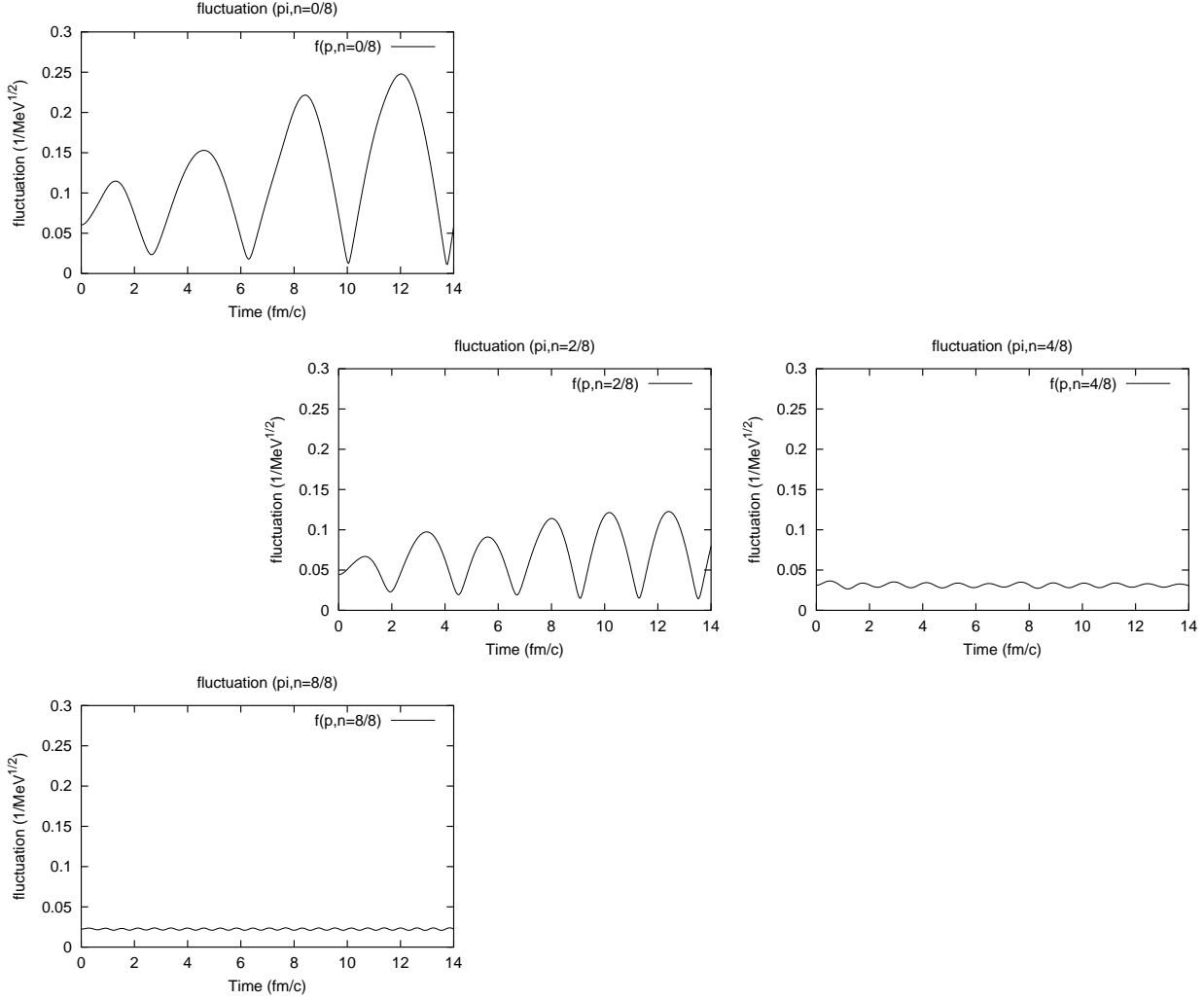
Fig. 2. The time-evolution of the fluctuation modes in the σ-direction are depicted. The figures of the upper-left, upper-right, lower-left and lower-right represent the time-evolution of the fluctuation modes with the quantum numbers $n^2 = 0$, $n^2 = 2^2$, $n^2 = 4^2$ and $n^2 = 8^2$, respectively.

Figure 1 shows us the time-evolution of the chiral condensate with $\overline{\varphi}(t = t_0 = 0) = 50$ MeV. It is shown that the behavior of the damped oscillation is realized. It should be noted that the center value of oscillation is slightly different from 92 MeV. The reason is that the effective potential $\mathcal{M}^{(0)}[\overline{\varphi}]$ contains the effects of quantum fluctuations in our approach. If the quantum fluctuation modes have no time-dependence, the value of condensate should be 92 MeV. It is seen in Fig.2 that the σ-modes oscillate. However, in Fig.3, it is seen that the amplitude of π-modes with low momenta ($n^2 \leq 2^2$) increases. Namely, these amplitude of π-modes with low momenta increases corresponding to the decreasing of the amplitude of the chiral condensate. However, the π-mode with high momenta are not amplified but oscillated only. This situation can be easily understood. The instability occurs when the time-

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Fig. 3. The time-evolution of the fluctuation modes in the π-direction are depicted. The figures of the upper-left, upper-right, lower-left and lower-right represent the time-evolution of the fluctuation modes with the quantum numbers $n^2 = 0$, $n^2 = 2^2$, $n^2 = 4^2$ and $n^2 = 8^2$, respectively.

dependent "mass" term $M_a(t)^2 = -m^2 + 12\lambda\overline{\varphi}_a(t) + \sum_{b \neq a} 4\lambda\overline{\varphi}_b(t) + \text{(quantum correction)}$ is negative and the absolute value of it is greater than $\boldsymbol{k}^2$. For σ-mode, the time-dependent mass is $M_0(t)^2 = -m^2 + 12\lambda\overline{\varphi}_0(t) + \text{(quantum correction)}$ as is similar to Eq.(3·9). However, for the π-mode, the mass $M_\pi(t)^2$ is less than $M_0(t)^2$ because $M_\pi(t)^2 = -m^2 + 4\lambda\overline{\varphi}_0(t) + \text{(quantum correction)}$. Thus, the instability occurs in the π-modes with low momenta. As a result, the behavior of the amplified oscillation is realized. On the other word, as was shown in §2, if the mode functions are expressed as $\eta_{\mathbf{k}}(t) = 1/\sqrt{2\epsilon_{\mathbf{k}}(t)}$, the instability is realized when $\epsilon_{\mathbf{k}}^2(t) < 0$. When $\epsilon_{\mathbf{k}}^2(t) < 0$ for $|\boldsymbol{k}| < \sqrt{-M^2}$, then $G(\boldsymbol{x}, \boldsymbol{x}, t)$ is obtained as

$$G(\boldsymbol{x}, \boldsymbol{x}, t) = \sum_{|\mathbf{k}| < \sqrt{-M^2}} \frac{-i}{2|\epsilon_{\mathbf{k}}(t)|} e^{\pm 2 \int^t |\epsilon_{\mathbf{k}}(t')| dt'} + \sum_{|\mathbf{k}| > \sqrt{-M^2}} \frac{1}{2\epsilon_{\mathbf{k}}(t)} . \tag{4·2}$$
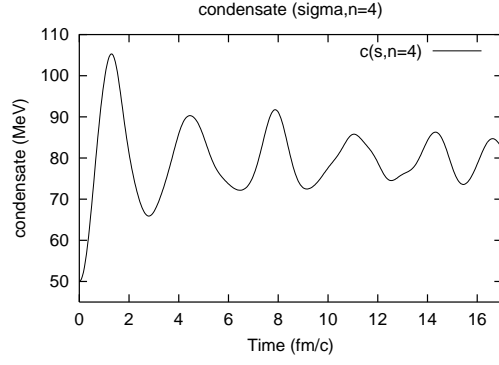
Fig. 4. The time-evolution of the chiral condensate is depicted in which the fluctuation modes are contained up to $n^2 = 4^2$.
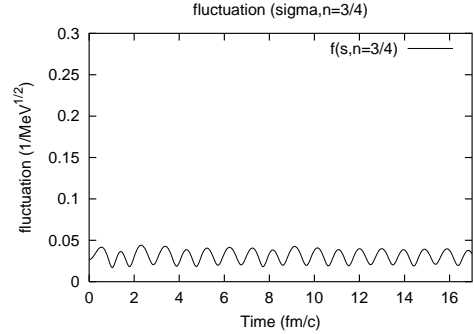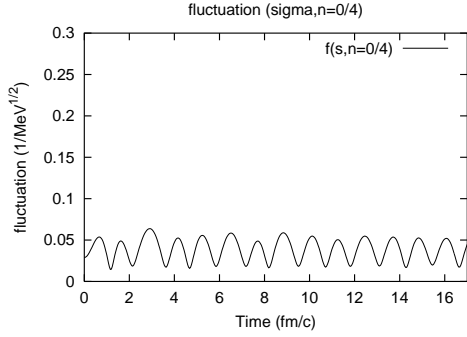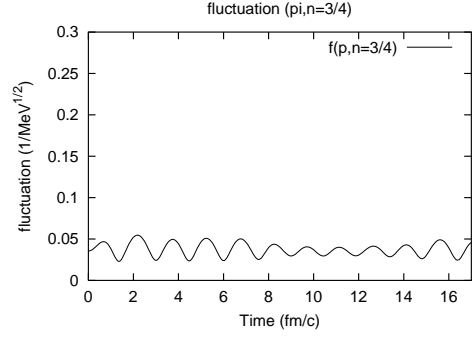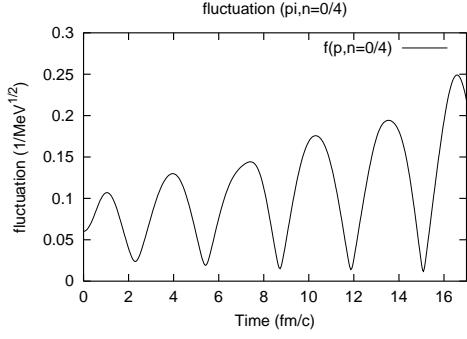

Fig. 5. The time-evolution of the fluctuation modes with the quantum numbers $n^2 = 0$ (left) and $n^2 = 3^2$ (right) are depicted in the σ-direction.

Namely, this means that the imaginary part appears in the equation of motion for the mean field $\overline{\varphi}_a(t)$. In this case the amplitudes of the low momentum modes with $|\boldsymbol{k}| < \sqrt{-M^2}$ of quantum fluctuation increase in time. Thus, the relaxation process for the chiral condensate occurs due to the instability of the quantum fluctuation. Then, the energy stored in the chiral condensate flows to the quantum fluctuations.

In Figs.4∼6 and 7∼9, the only a few fluctuation modes up to $n^2 = 4^2$ and $n^2 = 0$ are taken into account in each isospin direction, respectively. The time-evolution of the condensate, the fluctuation modes with σ-direction and the fluctuation modes with π-directions are depicted

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Fig. 6. The time-evolution of the fluctuation modes with the quantum numbers $n^2 = 0$ (left) and $n^2 = 3^2$ (right) are depicted in the π-direction.
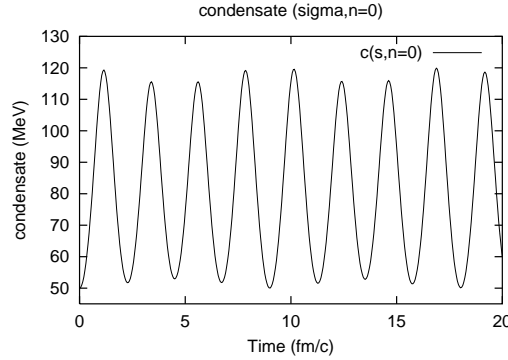
Fig. 7. The time-evolution of the chiral condensate is depicted in which the fluctuation mode with $n = 0$ is only contained.

in Figs.4, 5 and 6, respectively, up to $n^2 = 4^2$. The qualitative behavior is almost same as that in Figs.1~3. The fluctuation modes with $n^2 \geq 3^2$ for π-mode are not responsible for the damping of the amplitude of the condensate. However, in Figs.7~9, the qualitative behavior is quite different from Figs.1~3 and 4~6. The condensate is relaxed to the vacuum value no longer. The reason why the relaxation process is not realized is that the quantum fluctuation modes are not included sufficiently. Thus we conclude that the quantum fluctuation mode for π-direction with low momenta are responsible for the relaxation process of the chiral condensate during the chiral phase transition.

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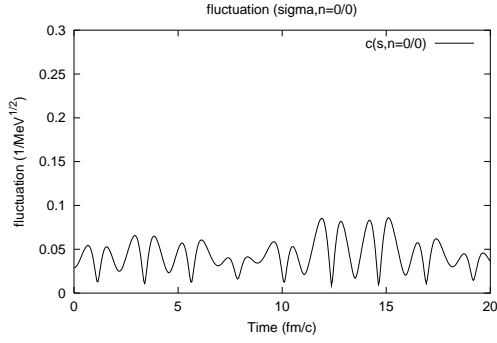
Fig. 8. The time-evolution of the fluctuation mode with $n = 0$ in the σ-direction is depicted.
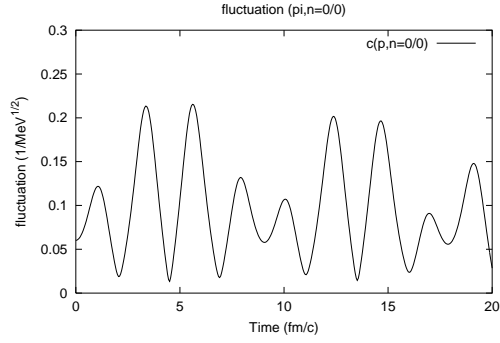

Fig. 9. The time-evolution of the fluctuation mode with $n = 0$ in the π-direction is depicted.
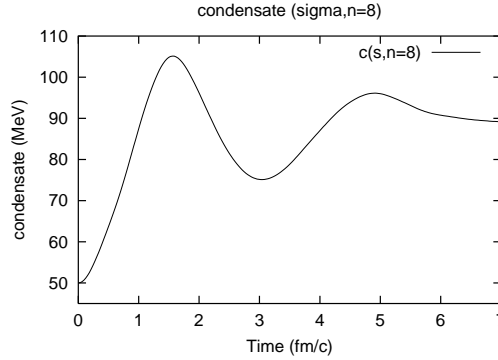

Fig. 10. The time-evolution of the chiral condensate on the expanding geometry is depicted in which the fluctuation modes are contained up to $n^2 = 8^2$.

Let us consider the expanding geometry[21] to break the time-reversal invariance. The basic equations are given in Appendix. The expanding parameter $a(t)$ is assumed so as to be proportional to time : $a(t) = t/\tau_0 + 1$. The parameter is taken as $\tau_0 = 1$ fm/c. The other parameters are same as those in Figs.1∼3. Also, it is assumed that the expansion occurs in $D = 3$-dimension. Figure 10 shows us that the behavior of the damped oscillation appears in the chiral condensate and the oscillation converges to the vacuum value. It is also seen that the amplitude of the quantum fluctuation modes also decreases gradually in time in Fig.11 (σ-modes) and Fig.12 (π-modes).

§5. Summary

In this paper, we have investigated the time-evolution of the chiral condensate and the quantum fluctuation around it in the context of the chiral phase transition from the symmetric to symmetry-broken phase. We have formulated the time-dependent variational method with the squeezed state for O(N) linear sigma model. It has been shown numerically that
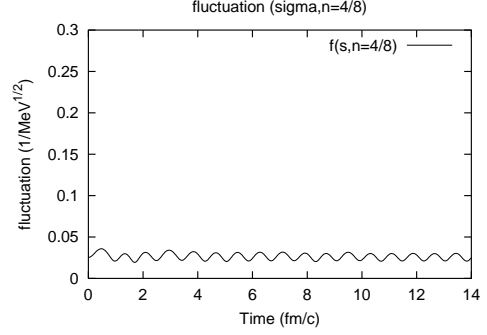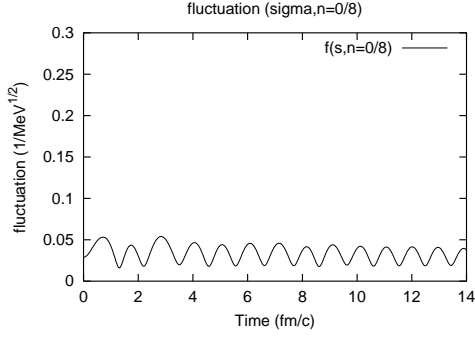
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Fig. 11. The time-evolution of the fluctuation modes with the quantum number $n^2 = 0$ (left) and $n^2 = 4^2$ (right) in the σ-direction on the expanding geometry are depicted.

the behavior like the damped oscillation for the chiral condensate appears due to the inclusion of the quantum fluctuations in the squeezed state approach. We have pointed out that the quantum fluctuations, especially π-modes with low momenta, are responsible for the occurrence of the relaxation process of the chiral condensate. Of course, the expansion of the system leads to a friction term.[8), 10)] However, even if the expansion is not taken into account, it is concluded that the relaxation process occurs due to the quantum fluctuations around the mean field configuration. It may be also interesting to investigate the implication to the parametric resonance in the π-mode.
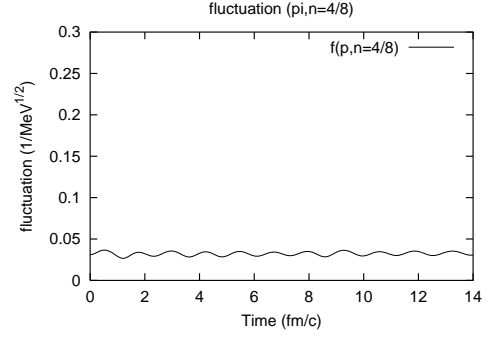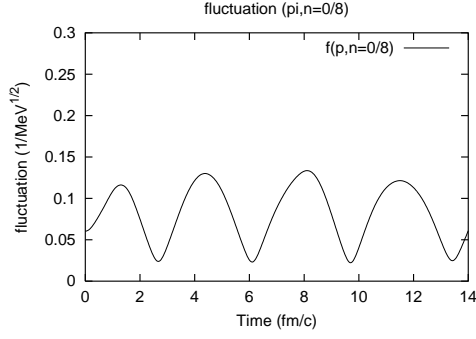
Acknowledgements

Fig. 12. The time-evolution of the fluctuation modes with the quantum number $n^2 = 0$ (left) and $n^2 = 4^2$ (right) in the π-direction on the expanding geometry are depicted.

Appendix A

—— *Expanding geometry* ——

The collisional region may expand spatially. If it is necessary to include the effect of the expanding geometry,[21] the metric may be taken as

$$ds^2 = dt^2 - a(t)^2 d\boldsymbol{x}^2 \quad = g_{\mu\nu}(t)dx^\mu dx^\nu \ , \tag{A·1}$$

where $a(t)$ represents the expansion of collisional region, which is governed by another dynamics. Here, $a(t)$ is given by hand. Hereafter, it is assumed that the expansion occurs in $D = 1$- or $D = 3$-dimension. Further, in $D = 1$, the direction of the expansion is adopted as the z-direction. The metric tensor $g_{\mu\nu}(t)$, its inverse and its determinant g are given by

$$g_{\mu\nu}(t) = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & -a(t)^2 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & -a(t)^2 \end{pmatrix} ,$$

$$g^{\mu\nu}(t) = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & -1/a(t)^2 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & -1/a(t)^2 \end{pmatrix},$$

$$g = \det g_{\mu\nu} = (-a(t)^2)^D \tag{A·2}$$

with $D = 1$ or 3. Then, equations of motion in Eq.(3·7) are modified because the d'Alembertian $\partial^2/\partial t^2 - \nabla^2$, which acts on $\overline{\varphi}_a$ and η^a, is replaced by

$$\begin{aligned} \overline{\varphi}_{a;\mu}^{\;;\mu} &= \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left(\sqrt{-g} g^{\mu\nu} \frac{\partial \overline{\varphi}_a}{\partial x^\nu} \right) \\ &= \frac{1}{\sqrt{-g}} \left[\frac{\partial}{\partial t} \left(\sqrt{-g} \frac{\partial \overline{\varphi}_a}{\partial t} \right) - \nabla \cdot \left(\sqrt{-g} \frac{1}{a(t)^2} \nabla \overline{\varphi}_a \right) \right] \\ &= \frac{\partial^2 \overline{\varphi}_a}{\partial t^2} + D \frac{\dot{a}(t)}{a(t)} \frac{\partial \overline{\varphi}_a}{\partial t} - \nabla_D^2 \overline{\varphi}_a . \end{aligned} \tag{A·3}$$

Namely, the d'Alembertian is replaced into

$$\frac{\partial^2}{\partial t^2} - \nabla^2 \longrightarrow \frac{\partial^2}{\partial t^2} + D \frac{\dot{a}(t)}{a(t)} \frac{\partial}{\partial t} - \nabla_D , \tag{A·4}$$

where the variable with suffix $D \ (= 1, 3)$ is defined as

$$f_D^2 = \frac{1}{a(t)^{D-1}} (f_x^2 + f_y^2) + \frac{1}{a(t)^2} f_z^2 . \tag{A·5}$$

As a result, the equations of motion we solve are modified from Eq.(3·7) as follows :

$$\ddot{\overline{\varphi}}_a(t) + D \frac{\dot{a}(t)}{a(t)} \dot{\overline{\varphi}}_a(t) - m^2 \overline{\varphi}_a(t) + 4\lambda \overline{\varphi}_a(t)^3 + 12\lambda \int \frac{d^3 \boldsymbol{k}}{(2\pi)^3} \eta_{\boldsymbol{k}}^a(t)^2 \cdot \overline{\varphi}_a(t)$$

$$+ 4\lambda \sum_{b \neq a} \left(\overline{\varphi}_b(t)^2 + \int \frac{d^3 \boldsymbol{k}}{(2\pi)^3} \eta_{\boldsymbol{k}}^b(t)^2 \right) \overline{\varphi}_a(t) - h\delta_{a0} = 0 ,$$

$$\ddot{\eta}_{\boldsymbol{k}}^a(t) + D \frac{\dot{a}(t)}{a(t)} \dot{\eta}_{\boldsymbol{k}}^a(t) + \left[k_D^2 - m^2 + 12\lambda \overline{\varphi}_a(t)^2 + 12\lambda \int \frac{d^3 \boldsymbol{k}'}{(2\pi)^3} \eta_{\boldsymbol{k}'}^a(t)^2 \right.$$

$$\left. + 4\lambda \sum_{b \neq a} \left(\overline{\varphi}_b(t)^2 + \int \frac{d^3 \boldsymbol{k}'}{(2\pi)^3} \eta_{\boldsymbol{k}'}^b(t)^2 \right) \right] \eta_{\boldsymbol{k}}^a(t) - \frac{1}{4\eta_{\boldsymbol{k}}^a(t)^3} = 0 . \tag{A·6}$$

Of course, the term with the first derivative of time causes the friction and the dissipative process is realized.

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